Solutions to Beam Deflections and Elastic Stability Problems

Civil Engineering Licensure Exam – Mock Exam

February 24, 2025

Problem 1: Maximum Deflection of a Simply Supported Beam with Central Point Load

A simply supported beam of span 6 m carries a point load of 20 kN at midspan. Determine the maximum deflection using the formula:

$$\delta = \frac{PL^3}{48EI}$$

where $EI = 50 \times 10^6\,\mathrm{N \cdot m^2}$.

Solution:

$$\begin{aligned} P &= 20 \times 10^3\,\mathrm{N} \\ L &= 6\,\mathrm{m} \\ EI &= 50 \times 10^6\,\mathrm{N \cdot m^2} \end{aligned}$$

$$\delta = \frac{20 \times 10^3 \times 6^3}{48 \times 50 \times 10^6} = 0.009\,\mathrm{m} = 9\,\mathrm{mm}$$

Reference: Deflection of Simply Supported Beam with Point Load

Problem 2: Maximum Deflection of a Cantilever Beam with Uniformly Distributed Load

A cantilever beam of length 3 m carries a uniformly distributed load of 5 kN/m. Determine the maximum deflection using:

$$\delta = \frac{5wL^4}{384EI}$$

where $EI = 40 \times 10^6\,\mathrm{N \cdot m^2}$.

Solution:

$$\begin{aligned} w &= 5 \times 10^3\,\mathrm{N/m} \\ L &= 3\,\mathrm{m} \\ EI &= 40 \times 10^6\,\mathrm{N \cdot m^2} \end{aligned}$$

$$\delta = \frac{5 \times 5 \times 10^3 \times 3^4}{384 \times 40 \times 10^6} = 0.0211\,\mathrm{m} = 21.1\,\mathrm{mm}$$

Reference: Deflection in Cantilever Beam with Uniform Load

Problem 3: Critical Buckling Load of a Pin-Ended Column

A column with an effective length of 4 m has a moment of inertia of $8 \times 10^6\,\mathrm{mm}^4$ and a modulus of elasticity of 200 GPa. Determine the critical buckling load.

Solution:

Using Euler's formula:

$$P_{\mathrm{cr}} = \frac{\pi^2 EI}{L^2}$$

$$\begin{aligned} L &= 4\,\mathrm{m} = 4000\,\mathrm{mm} \\ I &= 8 \times 10^6\,\mathrm{mm}^4 \\ E &= 200 \times 10^3 \times 10^6\,\mathrm{N/m^2} \end{aligned}$$

$$P_{\mathrm{cr}} = \frac{\pi^2 \times 200 \times 10^9 \times 8 \times 10^{-6}}{4^2} = 987.2\,\mathrm{kN}$$

Reference: Euler's Buckling Formula for Pin-Ended Columns

Problem 4: Deflection at Midspan of a Simply Supported Beam with Triangular Load

A simply supported beam of span 4 m carries a triangular load varying from zero at one end to 6 kN/m at the other end. Using the moment-area method, determine the approximate deflection at midspan.

Solution:

For a simply supported beam with a triangular load increasing to w_{max} at the end:

$$\delta_{\text{max}} = \frac{3w_{\text{max}}L^4}{20EI}$$

$$w_{\text{max}} = 6 \times 10^3 \, \text{N/m}$$
$$L = 4 \, \text{m}$$

Assuming $EI = 50 \times 10^6 \, \text{N} \cdot \text{m}^2$:

$$\delta_{\text{max}} = \frac{3 \times 6 \times 10^3 \times 4^4}{20 \times 50 \times 10^6} = 0.023 \, \text{m} = 23 \, \text{mm}$$

Reference: Triangular Load on a Simply Supported Beam

Problem 5: Slenderness Ratio of a Fixed-Free Steel Column

A steel column is fixed at one end and free at the other. If its effective length is 3 m and it has a radius of gyration of 50 mm, determine the slenderness ratio.

Solution:

The slenderness ratio λ is given by:

$$\lambda = \frac{L_{\text{eff}}}{r}$$

For a fixed-free column:

$$L_{\text{eff}} = 2L$$

$$L = 3 \, \text{m} = 3000 \, \text{mm}$$
$$r = 50 \, \text{mm}$$

$$\lambda = \frac{2 \times 3000}{50} = 120$$

Reference: Slenderness Ratio Calculation for Columns